Appendix

## Flowchart of Solution Algorithm

The flowchart of algorithm 2 is presented as Fig. 1.



Fig. 1 Flowchart of the SOC-based C&CG algorithm combined with SBT procedure

## Topology of the Modified 39-bus and 20-node IEHS



Fig. 2 Topology of the modified 39-bus and 20-node IEHS

The topology of small-scale test IEHS is displayed as Fig. 2. The test IEHS is composed of two city-level DHSs and a provincial-level PTS. The components painted red denote the candidate components and they are ranked for the installation illustration. Specifically, the existing DHS-1 is extended and DHS-2 is newly installed to test the effectiveness of proposed planning model. Note that Heat nodes 14, 15 and 20 are connecting heat nodes, which are selected to be connected to deliver heat power.



Fig. 3 5-block load level and wind power profile

Based on the k-means clustering method [24], the base-scenario 5 blocks of load level and wind power as Fig. 3 are derived from data in [4] to reflect the seasonal characteristics of the electric/heat load demands and wind power. The 1st load block represents the winter load situation where heat load is the highest. The 3rd load block denotes the summer situation where electric load is the highest while the heat load is very low. At the 5th load block, the available wind power is high while both the electric and heat loads are low. The 1st, 3rd and the 5th  load blocks represent most critical operation status, which may affect the reliability of the energy supply.

## Effectiveness of Proposed Reformulation Method

To demonstrate the effectiveness of SBT procedure in tightening the QC relaxation of DHS model, part of the optimized/calculated heat power and mass flow rates of case 1(direct) and case 1(SBT) are presented as Fig. 4-Fig. 7, respectively.



Fig. Optimized/calculated mass flow of supply pipelines in case1 (direct)



Fig. Optimized/calculated mass flow of supply pipelines in case1 (SBT)



Fig. Optimized/calculated heat power of supply pipelines in case1 (direct)



Fig. Optimized/calculated heat power of supply pipelines in case1 (SBT)

The mass flow and heat power of pipelines in DHS-2 are obtained according to the loads of load block 1 in 10th planning year. Note that the optimized mass flow rates/heat power are obtained directly from the optimization of reformulated planning model. While the calculated mass flow rates/heat power are computed from the values of state variables. (*i.e.* thecalculated mass flow is computed according to the values of nodal pressure as equation). As Fig. 4-Fig. 7 shown, there is obvious error between the optimized values and the calculated values in case 1 (direct). While the application of SBT procedure can effectively reduce the relaxation gap between the optimized values and the calculated values in case 1 (SBT). However, the values of optimized heat power/mass flow of case 1 (direct) and case 1 (SBT) are quiet close. Since the investment decisions of candidate assets in DHS are mainly determined by the values of mass flow and heat power, the investment decisions of case 1 (direct) and case 1 (SBT) are the same. But the values of state variables in case 1 (direct) are not accurate. Thus, the combination of SBT procedure and the proposed reformulation method is more favorable to obtain the precise state variables and operation costs.

To further investigate the effectiveness of proposed reformulation method for DHS, the numerical comparisons of proposed method and other solution methods are carried out. As reviewed in introduction, the existing solution methods in [4], [14]-[19] are all not suitable for the planning model of IEHS because they all require the fixed DHS topology. Thus, the numerical results of the optimal heat flow (OTF) problem obtained by different methods are compared for illustration. The following aspects are worthy of attention:

* OTF is tested based on the determined topology of DHS-2 with the heat load at 1st load block in the first year.
* The results obtained by solution methods in [4], [16], [17], [19] are used for comparison. Since the method in [15] assumed most fluid constant and method in [18] simplified part of constraints associated with temperature variables, these two methods are not compared. For ease of illustration, the methods in [4], [16], [17] and [19] are respectively represented by “Iteration”, “Decom”, “SLP” and “SOC”. Detailed solution procedures can be found in [4], [16], [17] and [19]. “Proposed” represents the combination of proposed reformulation method and the SBT procedure.
* The “Iteration” method and the “SLP” method need an initial set of feasible decision variables. Since it is not easy to get initial sets directly, the initial sets of feasible decision variables are modified from the values obtained by the proposed method.

TABLE I

Comparison of Different Solution methods to OTF

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Method | Iteration | Decom | SLP | SOC | proposed |
| Cost ($) | 535.7282 | 549.4452 | 534.4832 | 547.09 | 534.4193 |
| Time(s) | 2.41 | 1.73 | 2.65 | 6.30 | 2.33 |

TABLE I presents the computation time and the costs of OTF solved by different methods. Moreover, the supply/return temperatures of mixtures at each node and heat power/fluid within pipelines are displayed in Fig. 8-Fig. 11, respectively. No matter the values of temperature and mass flow variables or the costs of OEF, the results of the “Iteration” method, the “SLP” method and the proposed method are very analogous. The comparisons of objective functions and the values of state variables illustrate that the proposed method can achieve good accuracy in solving the optimization problem of DHS. Since the pressure loss constraints and the bounds of fluid were not considered in the “Decom” method, the mass flow rates obtained by the “Decom” method may be too large. Compared with proposed method, two more sets of auxiliary variables are introduced to represent heat quantities in “SOC” method. To convexify the optimization model, these two sets of auxiliary variables are conically relaxed. Thus, more relaxation errors may exist in the “SOC” method.

Without the bounds of mass flow rates, the “Decom” method cannot identify whether the heat pipelines should be expanded in the planning model. Both the “Iteration” method and “SLP” method require an initial set of feasible decision variables. The initial set of feasible decision variables cannot be obtained directly in the planning model of IEHS due to the alternative topology of DHS. In “SOC” method, the topology of DHS should also be determined in advance to get the initial sets of variables for the solving procedure. As such, these four methods are all not suitable for the planning model of IEHS. Compared with these methods, the proposed method can not only obtain the accurate results of the DHS optimization problem, but also can tackle the planning problem of IEHS with changeable topology flexibly.



Fig. Supply temperatures of mixtures at nodes in decided DHS-2 (OTF)



Fig. 9 Return temperatures of mixtures at nodes in decided DHS-2 (OTF)



Fig. Mass flow rates within supply pipelines of decided DHS-2 (OTF)



Fig. 11 Heat power within supply pipelines of decided DHS-2 (OTF)

## Effectiveness of SOC-based C&CG Algorithm

The energy outputs of different cases at 10th planning year are presented in consideration of different energies, as shown in Fig. 12, Fig. 13. Note that the loads of case 1 is the forecast values of load blocks. The loads of cases 2-3 are the worst-case scenarios identified by the C&CG algorithm. It can be clearly seen that the electric/heat loads of cases 2-3 are higher than forecast values of case 1 while the available wind power of cases 2-3 is lower than case 1. In case 2, due to the lower available wind power and higher electric/heat loads, CHP unit C4 is installed instead of EB B2 to produce electricity and heat energy simultaneously. In the worst-case scenarios of case 3, CHP unit C6 fails at 1st and 4th load blocks, existing non-CHP thermal generator at bus 39 fails at 3rd load block. Thus, EB B2 and B3 have to be installed to supply heat power at 1st load block. More non-CHP thermal generator G3-G5 are constructed to satisfy electric demands at 3rd load block.



Fig. Electric outputs of the devices at the 10th planning year in cases 1-3



Fig. Heat outputs of the devices at the 10th planning year in cases 1-3

## Practical-scale IEHS

The proposed RO planning model and solution methods are further applied to a real IEHS in Northeastern China which composed of a provincial-level PTS and 5 city-level DHSs [4]. The existing components in real test system include 16 non-CHP thermal generators, 22 wind farms, 381 transmission lines and 25 heat pipelines. Candidate assets comprise of 20 non-CHP thermal generators, 17 CHP units, 85 transmission lines and 18 pipelines. A 10-year planning horizon is carried out with the same 5 load blocks as small-scale IEHS in each year. Peak electric load, heat load in the first planning year are respectively 9950.95 MW and 2057 MW. Other parameters are the same as the small-scale IEHS.

TABLE

Average Mass Flow error/Heat Quantity Error of different cases

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Case | Case 1 | | Case 2 | | Case 3 | |
| Direct | SBT | C&CG | C&CG  +SBT | C&CG | C&CG  +SBT |
| fluid error | 25.22% | 2.79% | 25.44% | 2.58% | 25.46% | 2.67% |
| source error | 1.05% | 10-4% | 1.23% | 10-4% | 1.21% | 10-4% |
| load error | 7.59% | 0.31% | 5.06% | 0.28% | 7.73% | 0.28% |
| Pipeline error | 8.22% | 0.13% | 6.18% | 0.12% | 5.91% | 0.12% |
| IC(B$) | 2.0176 | 2.0168 | 2.3005 | 2.2991 | 2.3310 | 2.3310 |
| OC(B$) | 4.8882 | 4.8883 | 5.7284 | 5.7309 | 5.7374 | 5.7554 |
| iteration | - | 2(SBT) | 3 | 3 | 8 | 8 |
| Time(s) | 1359 | 2547.4 | 10942 | 19931 | 57376 | 88448 |

TABLE

Comparison of Planning Results in Cases 1-3

|  |  |  |  |
| --- | --- | --- | --- |
| Assets | Case 1 | Case 2 | Case 3 |
| Non-CHP | G2,8,G3,6,G6,7,G7,3,  G10,5,G11,9,G12,9,  G13,8,G16,1,G17,10  G18,2,G20,2 (12) | G1,6,G2,9,G3,3,G4,8,  G5,10,G6,4,G7,1,G9,10  G10,5,G13,5,G14,10,  G16,1,G17,7  G18,1,G20,1 (15) | G1,8,G2,9,G3,1,G4,4,  G5,10,G6,3,G7,1,G10,9,  G12,9,G13,5,G14,10,  G16,1,G17,7  G18,1,G20,1 (15) |
| CHP | C7,1,C9,1,C10,1,C11,1  C12,1,C13,1,C14,1,  C15,1,C16,1,C17,1 (7) | C6,1,C8,1,C10,1,C11,1  C12,1,C13,1,C14,1,  C15,1,C16,1,C17,1 (7) | C6,1,C8,1,C10,1,C11,1  C12,1,C13,1,C14,1,  C15,1,C16,1,C17,1(7) |
| Line | L2,1,L3,1,L4,1,L5,1,  L6,1,L8,1,L10,1,L13,1  L16,1,L17,1,L30,1,  L56,1,L58,1,L64,1  L67,1,L69,1,L70,1  L71,1,L77,1,L80,1 (20) | L2,1,L3,1,L4,1,L5,1,  L6,1,L8,1,L10,1,L13,1  L16,1,L17,1,L30,1,L40,1  L56,1,L58,1,L64,1  L67,1,L69,1,L70,1  L71,1,L77,1,L80,1  (21) | L2,1,L3,1,L4,1,L5,1,  L6,1,L8,1,L10,1,L13,1  L16,1,L17,1,L30,1,L36,1  L37,1,L40,1,L43,1  L56,1,L58,1,L64,1  L67,1,L69,1,L70,1  L71,1,L77,1,L80,1  (24) |
| Pipeline | P3,1,P4,1,P7,1,P8,1,  P10,1,P11,1,P13,1P14,1,  P17,1,P18,1 (10) | P3,1,P4,1,P7,1,P8,1,  P10,1,P11,1,P13,1P14,1,  P17,1,P18,1 (10) | P3,1,P4,1,P7,1,P8,1,  P10,1,P11,1,P13,1P14,1,  P17,1,P18,1 (10) |

TABLE

Comparisons of planning results of cases 1-2 with/without SBT

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Investment cost (B$) | Case 1 (direct) | Case 1 (SBT) | Case 2 (C&CG) | Case 2 (C&CG+SBT) |
| Non-CHP | 0.8498 | 0.8490 | 1.0654 | 1.0640 |
| CHP | 0.9394 | 0.9394 | 1.0002 | 1.0002 |
| Line | 0.1441 | 0.1441 | 0.1508 | 0.1508 |
| Pipeline | 0.0841 | 0.0841 | 0.0841 | 0.0841 |
| Difference | G10,7,G13,6G10,5,G13,8 | | G12,10,G14,8,G17,9  G9,10,G14,10,G17,7 | |

The same three cases are explored for real IEHS. The average mass flow error and the average heat quantity errors of different components are shown in TABLE II. And the detailed planning results are displayed in TABLE III. It can be seen that both the average mass flow error and the average heat quantity errors are reduced evidently by applying the SBT procedure. In practical IEHS, the application of SBT procedure leads to the different planning schemes of cases 1-2. The comparisons of different planning results are shown in TABLE IV. Though SBT procedure reduces the relaxation errors of DHS constraints, it affects the construction of non-CHP thermal generators. It is because the utilization of SBT procedure can obtain more accurate heat power production of CHP units. Since there is a strong relationship between electricity output and heat energy production of CHP units, the electricity output of CHP units is also refined by SBT procedure. Thus, the tightening procedure of relaxations for DHS affects the electricity generation and the installation of non-CHP thermal generators. It also illustrates the strong correlation between DHS and PTS. The combination of proposed convex reformulation method and SBT procedure can obtain the accurate investment decisions and operation costs for the planning model.

For case 1, the computation time of SBT procedure is nearly double of solving the reformulated planning model directly. It is because SBT procedure needs 2 iterations to be converged. In cases 2-3, the complexity of master planning problem improves with the iterations of C&CG algorithm. The computation time is therefore increased a lot. As well recognized in [22], it is still an issue for applying the robust optimization to the planning model of practical large-scale system. However, the running time less than 24 hours is still acceptable for a practical system with 10-year planning horizon. (In [31], it spends several weeks for analyzing a single future scenario in NYISO practice.)

Similar as small-scale IEHS, the consideration of uncertainties lead to more conservative investment decisions. To satisfy the electric load in worst-case scenarios, 3 more non-CHP thermal generators G1, G4, G5 and 1 more transmission line L40 are installed in case 2. Meanwhile, CHP units C6 and C8 with larger capacity is installed instead of C7 and C9 to meet heat demands. When uncertainties of possible component outage is further considered in case 3, all of large-capacity generators are constructed ahead and three more transmission lines L36, L37, L43 are installed to prevent load shedding in worst-case scenarios. Since electric and heat load increases in worst-case scenarios, the operation costs of cases 2-3 also increases. In summary, the simulation results verify that the proposed SOC-based C&CG algorithm combined with SBT procedure can obtain a reliable IEHS against the various uncertainties of electric/heat load, wind power and N-1 criterion of components.

1. New York Independent System Operator, “2015 congestion assessment and resource integration study, CARIS-Phase I,” Nov. 2015 [Online]. Available:http://www.nyiso.com/public/webdocs/markets\_operations/services/planning/Planning\_Studies/Economic\_Planning\_Studies\_(CARIS)/CARIS\_Final\_Reports/2015\_CARIS\_Report\_FINAL.pdf.